Solutions of JEE Advanced-2 | Paper–2 | 12th Pass | JEE 2024

PHYSICS

SECTION - 1 | SINGLE CHOICE CORRECT TYPE

1.(B) Let us consider a spherical surface of radius r and thickness dr.

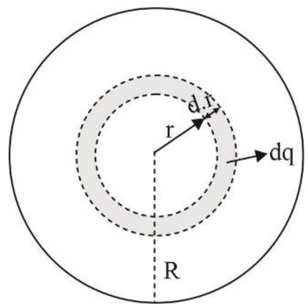
$dq = \rho dv$

$\Rightarrow\ dq = 4\pi r^2 dr \dfrac{1}{r} = 4\pi r dr$

$dq = 4\pi r dr$

Change in the volume of radius r

$q = \int dq = 4\pi \int_0^r r dr$



$q = \dfrac{4\pi r^2}{2}$

$q = 2\pi r^2$

By gauss law, electric field inside

$E_{in}(4\pi r^2) = \dfrac{q_{enclosed}}{\epsilon_0}$

$E_{in}(4\pi r^2) = \dfrac{2\pi r^2}{\epsilon_0}$

$E_{in} = \dfrac{1}{2\epsilon_0}$

So electric field inside the sphere is constant

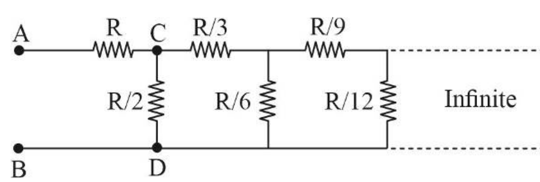
By gauss law electric field outside

$E_{out} 4\pi r^2 = \dfrac{q_{enclosed}}{\epsilon_0}$

$E_{out} 4\pi r^2 = \dfrac{2\pi R^2}{\epsilon_0}$

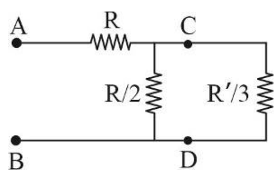
$E_{out} = \dfrac{R^2}{2\epsilon_0}\dfrac{1}{r^2} \Rightarrow\ E_{out} \propto \dfrac{1}{r^2}$

2.(D) Let R_{eq} between A and B is R'



So, R_{eq} between C and D will be $\dfrac{R'}{3}$

So,



$$(R_{eq})_{AB} = R + \frac{\frac{R}{2} \cdot \frac{R'}{3}}{\frac{R}{2} + \frac{R'}{3}}$$

$$R' = R + \frac{RR'/6}{\frac{3R+2R'}{6}}$$

$$R' = R + \frac{RR'}{(3R+2R')}$$

$$R' = \frac{3R^2 + 2RR' + RR'}{(3R+2R')}$$

$$3RR' + 2R'^2 = 3R^2 + 3RR'$$

$$R' = \sqrt{\frac{3}{2}}R$$

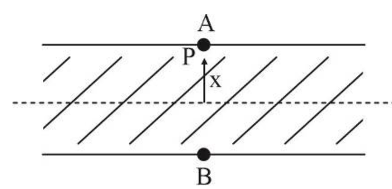
3.(B) $E_P = \frac{\rho x}{\epsilon_0}$ where x is distance from central plane.

So, if there is a charge q at point p the force on the charge particle is $F_P = qE_P$

$$\vec{F}_P = \frac{-q\rho\vec{x}}{E_0} \quad \ldots\text{(i)}$$

$$\vec{F}_P \propto -\vec{x}$$

So the charge $-q$ will execute SHM



$$\vec{F}_{SHM} = -m\omega^2\vec{x} \quad \ldots\text{(ii)}$$

(i) = (ii)

$$\frac{q\rho}{\epsilon_0} = m\omega^2$$

$$\omega = \sqrt{\frac{q\rho}{m\epsilon_0}}$$

$$T = 2\pi\sqrt{\frac{m\epsilon_0}{q\rho}}$$

So, the time from A to B, will be $\frac{T}{2}$

So, time from A to B, is $\pi\sqrt{\frac{m\epsilon_0}{q\rho}}$

4.(A) Torque on dipole

$$\tau = pE\sin\theta$$

It θ is small $\sin\theta \simeq \theta$

$$\tau = pE\theta$$

$$I\alpha = pE\theta$$

$$\alpha = \frac{pE}{I}\theta \qquad \ldots(i)$$

For SHM $\alpha = \omega^2\theta \qquad \ldots(ii)$

(i) = (ii)

$$\omega = \sqrt{\frac{pE}{I}} \qquad \ldots(iii)$$

$$p = q(2R) \qquad \ldots(iv)$$

$$I = MR^2 + m_1R^2 + m_2R^2$$

$$I = R^2[M + m_1 + m_2]$$

$$\omega = \sqrt{\frac{2qRE}{R^2(M + m_1 + m_2)}}$$

$$\omega = \sqrt{\frac{2qE}{R(M + m_1 + m_2)}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R(M + 2m_1 + 2m_2)}{2qE}}$$

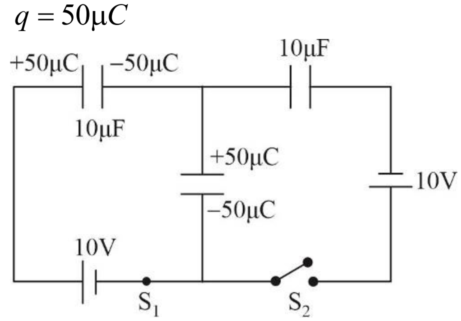
SECTION 2 | MULTIPLE CORRECT ANSWERS TYPE

5.(BCD) When S_1 is closed for long time

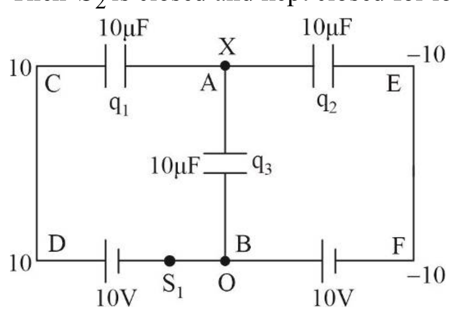
$$q = C_{eq}V$$

$$q = \left(\frac{10 \times 10}{10 + 10}\right)10$$

$$q = 50\mu C$$



Then S_2 is closed and kept closed for long time



Let potential of point B is zero and let potential of point A is x.

At junction A :

$q_1 + q_2 + q_3 = 0$

$(x-10)10 + [x-(-10)]10 + (x-0)10 = 0$

$10x - 100 + 10x + 100 + 10x = 0$

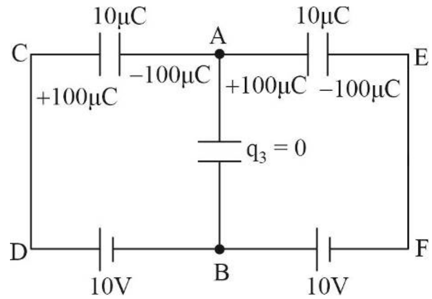
$x = 0$

So, the potential difference points A and B is zero

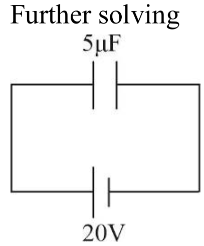
So, charge $q_3 = (x-0)10\mu C$

$= (0-0) \times 10 = 0$

Final situation after S_2 is kept closed for long time



Further solving



$q = C_{eq}V$

$q = 5 \times 20$

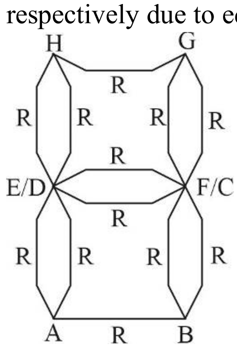
$q = 100\mu C$

The charge passed from point A after closing S_2 is $100\mu C$

6.(AC) Here $V_E = V_D$

$V_F = V_C$

So, wire ED and FC will be removed. Then points E and D and points F and C will be connected respectively due to equal potential.



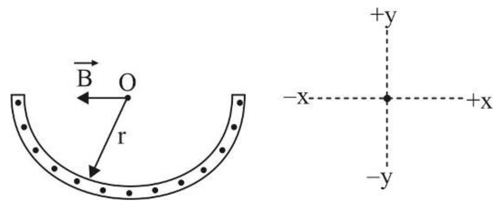
Solving R_{eq} between (A and B) $= \frac{7R}{12}$

As potential of points E and D is same, so no current will pass through wire ED.

7.(BC) Magnetic field due to thin walled semicircular plane $= B = \dfrac{\mu_0 I \sin\left(\frac{\theta}{2}\right)}{\pi\theta r}$ $(\theta = 180°) = \pi$

$$= \frac{\mu_0 I \sin\left(\frac{\pi}{2}\right)}{\pi^2 r}$$

$$\vec{B} = \frac{\mu_0 I}{\pi^2 r}(-\hat{i})$$



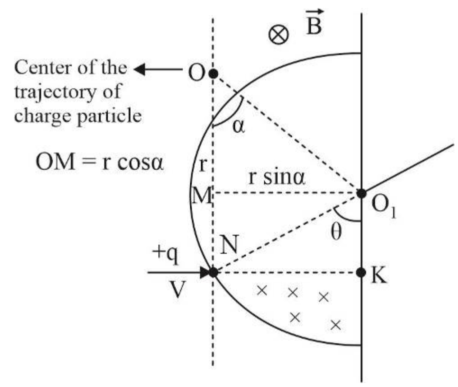
$\dfrac{\pi}{2}$ angle is there between the $\vec{B}$ and $\vec{v}$ of charge $+q$ at O, so radius of curvature at $O = \dfrac{mv}{qB} = \dfrac{mv\pi^2}{q\mu_0 I}$

8.(BD) $OM = r\cos\alpha$

$O_1K = MN = r - r\cos\alpha$

$ON = r$

$OO_1 = r$



Given $r\sin\alpha = \dfrac{mv}{2qB}$

$$r\sin\alpha = \frac{r}{2}$$

$$\sin\alpha = \frac{1}{2}$$

$$\alpha = 30°$$

$$\tan\theta = \frac{NK}{O_1K} = \frac{r\sin\alpha}{r - r\cos\alpha}$$

$$\tan\theta = \frac{r\sin 30°}{r - r\cos 30°}$$

$$\tan\theta = \frac{1/2}{1 - \frac{\sqrt{3}}{2}} = \frac{1}{\left(2-\sqrt{3}\right)}$$

Distance travelled in the magnetic field

$$= 2\pi r\frac{\alpha}{360°}$$

$$= 2\pi\frac{mv}{qB}\frac{30}{360}$$

$$= 2\pi\frac{mv}{qB}\times\frac{1}{12} = \frac{\pi mv}{6qB}$$

9.(BCD) $F_{net} = Ma_{CM}$

$F_1 + F_2 + f_s = M\,a_{CM}$

$24 + 3t + f_s = Ma_{CM}$ ...(i)

$\Rightarrow$ $\tau_{net} = I\alpha$ [about CM of disc]

$$F_1R - f_sR - F_2\frac{R}{2} = \frac{MR^2}{2}\alpha$$

$$R\left(24 - f_s - \frac{3t}{2}\right) = \frac{MR^2\alpha}{2} \qquad \ldots(ii)$$

For pure rolling

$$a_{CM} = R\alpha \qquad \ldots(iii)$$

By (ii) and (iii)

$$R\left(24 - f_s - \frac{3t}{2}\right) = \frac{MR^2}{2}\frac{a_{CM}}{R}$$

$$\left(24 - f_s - \frac{3t}{2}\right) = \frac{Ma_{CM}}{2} \qquad \ldots(iv)$$

$$24 + 3t + f_s = Ma_{CM} \quad \ldots(i)$$

Solving (i) and (iv)

$$48 + \frac{3t}{2} = \frac{3}{2}Ma_{CM}$$

$$\frac{2(48)}{3} + \frac{2}{3}\,\frac{3}{2}t = Ma_{CM}$$

$$32 + t = Ma_{CM} \qquad \ldots(v)$$

By (i) and (iv)

$$24 + 3t + f_s = 32 + t$$

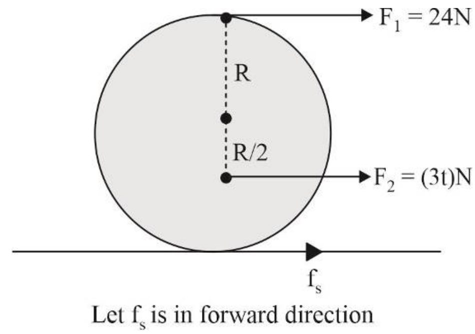
$$f_s = 8 - 2t \qquad \ldots(vi)$$

at $t = 1\text{sec}, M = 1kg$

$$a_{CM} = 33m/s^2$$

at $t = 4\text{sec}$

$$f_s = 0$$



For translational equilibrium

10.(BC) $N_2 = f_s \qquad \ldots(i)$

$\Rightarrow M_1g + Mg + M_2g = N_1 \quad \ldots(ii)$

$\Rightarrow$ for equilibrium

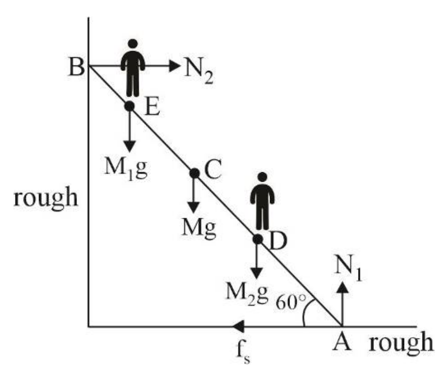
$$(\vec{\tau}_{net})_A = 0$$

$$N_2 l\sin 60° = Mg\frac{l}{2}\cos 60° + M_1gAE\cos 60° + M_2gAD\cos 60°$$

$$\frac{N_2 l\sqrt{3}}{2} = \frac{g}{2}\left[\frac{Ml}{2} + M_1AE + M_2AD\right]$$

If $AD = \frac{l}{4}$ and $AE = \frac{3l}{4}$

$$\frac{N_2 l\sqrt{3}}{2} = \frac{g}{2}\left[\frac{Ml}{2} + M\frac{3l}{4} + M\frac{l}{4}\right]$$



$N_2 l\sqrt{3} = g\dfrac{(2Ml + 3Ml + Ml)}{4}$

$N_2 l\sqrt{3} = g\dfrac{6Ml}{4}$

$N_2 = \dfrac{3Mg}{2\sqrt{3}} = \dfrac{\sqrt{3}Mg}{2}$

$\Rightarrow f_s = N_2 = \dfrac{\sqrt{3}Mg}{2}$

By changing the value of x_1 N_1 will not change due to expression (ii).

SECTION 3 | SINGLE DIGIT INTEGER TYPE

1.(2) For hemisphere $E_{center} = \dfrac{GM}{2R^2}$

So, value of x is 2

2.(3) The potential at the center of ring

$V_1 = \dfrac{-GM}{R}$

The potential at the center of disc is

$V_2 = \dfrac{-2GM}{R}$

Net potential point O $= V_1 + V_2$

$= \dfrac{-GM}{R} - \dfrac{2GM}{R}$

$= \dfrac{-3GM}{R}$

W to bring a mass m from infinity to point $O = \left[(V_{net})_f - (V_{net})_i\right]m$

$= \left[\left(\dfrac{-3GM}{R}\right) - 0\right]m$

$W = \dfrac{-3GMm}{R}$

So, the value of x is 3.

3.(4) E_{due} to $arc = \dfrac{\lambda \sin 30}{2\pi\varepsilon_0 R} = \dfrac{\lambda}{4\pi \in_0 R}$

$F_E = mg$

$qE = mg$

$\dfrac{q\lambda}{4\pi \in_0 R} = mg$ …(i)

$\lambda = \dfrac{q}{2\pi R / 6}$ [as $\theta = 60°$]

$\lambda = \dfrac{3q}{\pi R}$ …(ii)

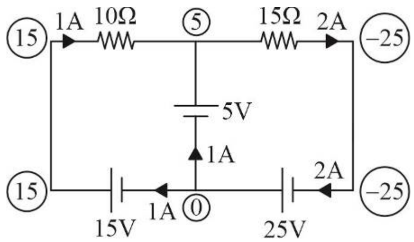
Put value of λ in (i)

$$\frac{q}{4\pi \in_0 R}\frac{3q}{\pi R} = mg$$

$$Q = \frac{4\pi^2 R^2 \in_0 mg}{3q}$$

So, value of x is 4.

4.(2)



So current in $10\Omega = \frac{15-5}{10} = \frac{10}{10} = 1A$

Current in $15\Omega = \frac{5-(-25)}{15} = \frac{30}{15} = 2A$

So current through $5V$ cell is 1A

$P_1 \rightarrow$ power through $5V$ cell $= VI$

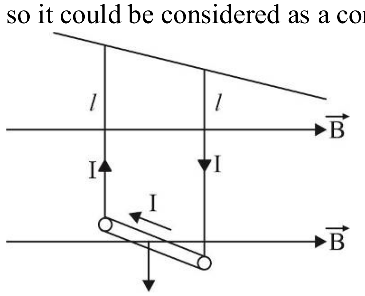
$= 5 \times 1 = 5W$

$P_2 \rightarrow$ power through 10Ω resistor $= I^2 R$

$= 1^2(10) = 10W$

$\frac{P_1}{P_2} = \frac{5}{10} = \frac{1}{2} \qquad \Rightarrow \qquad y = 2$

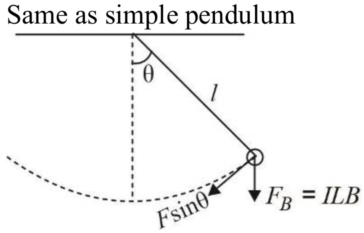
5.(2) As magnetic force will have same magnitude and same vertically downward direction at every point, so it could be considered as a conservative force



$F_B = ILB\sin 90°$

$F_B = ILB$

Same as simple pendulum



$\tau = lF\sin\theta$

$I'\alpha = lF\sin\theta$

$I'\alpha = l[ILB]\sin\theta$

$\alpha = \dfrac{lLIB\sin\theta}{I'}$

If $\theta \to$ small $\quad \sin = \theta$

$\alpha = \dfrac{lLIB}{I'}\theta \qquad \ldots(i)$

For SHM $\alpha = \omega^2\theta \qquad \ldots(ii)$

Comparing (i) & (ii)

$\omega = \sqrt{\dfrac{lLIB}{I'}}$

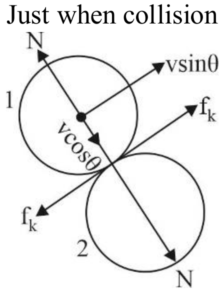
$T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{I'}{lLIB}}$

Value of x is 2.

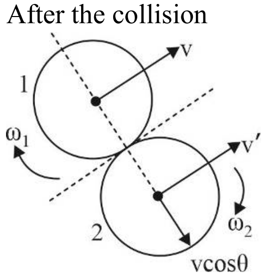
6.(2) For elastic collision the component of velocities along the common normal will inter change as the two balls are identical

But here the balls are rough so friction force will also act between the balls during collision

Just when collision starts



After the collision



Along the common normal for ball (2)

$\int N dt = \Delta p = mv\cos\theta \qquad \ldots(i)$

Along the common tangent apply conservation of linear and angular momentum

$mv\sin\theta = mv' + mv'$

$v' = \dfrac{v\sin\theta}{2} \qquad (2)$

For ball (2)

$R\int f_k dt = I\omega_2$

$R\mu\int N dt = I\omega_2 \qquad$ [From (1)]

$\mu mv\cos\theta = \dfrac{2}{5}mR\omega$

$\omega = \frac{5}{2}\frac{\mu v \cos\theta}{R}$

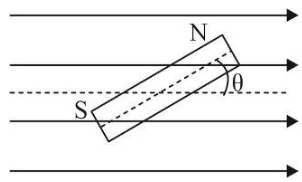
So, value of x is 2.

7.(2) $\tau = MB\sin\theta$

$I\alpha = MB\sin\theta$

$\frac{ml^2}{12}\alpha = MB\sin\theta$

If $\theta \to \text{small}$ $\quad \sin\theta = \theta$



$\frac{ml^2}{12}\alpha = MB\theta$

$\alpha = \frac{12MB\theta}{ml^2}$ ...(i)

For SHM

$\alpha = \omega^2\theta$ ...(ii)

(i) = (ii)

$\omega = \sqrt{\frac{12MB}{ml^2}}$

$T = \frac{2\pi}{\omega}$

$T = 2\pi\sqrt{\frac{ml^2}{12MB}}$

Value of x is 2

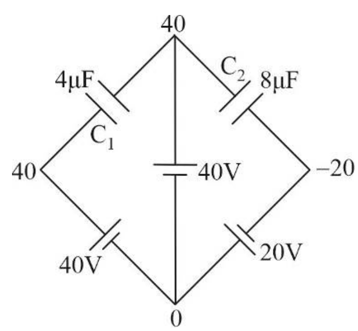
8.(0) Charge on $4\mu F$ is $Q_1 = CV$

$Q_1 = (4\mu F)0 = 0$

Charge on $8\mu F$ is $Q_2 = CV$

$= 8\mu F(40-(-20))$

$= 8\mu F 60 = 48\mu C$



$\frac{U_1}{U_2} = \frac{Q_1^2/2C_1}{Q_2^2/2C_2} = 0$

As $Q_1 = 0$

CHEMISTRY

SECTION - 1 | SINGLE CHOICE CORRECT TYPE

1.(C) Clean water has BOD value of < 5 while polluted water has BOD 15 or more.

2.(A) $I^- + H_2O_2 \longrightarrow \underset{(A)}{I_2} + H_2O$

$I_2 + \underset{(\text{Indicator})}{\text{Starch}} \longrightarrow \text{Blue}$

3.(D) $MX : K_{sp} = S^2 = 4\times10^{-8}$

$S = 2\times10^{-4}$

$MX_2 : K_{sp} = 4S^3 = 3.2\times10^{-14} \Rightarrow S = 2\times10^{-5}$

$M_3X : K_{sp} = 27S^4 = 2.7\times10^{-15} \Rightarrow S = 10^{-4}$

Order of solubility is $MX > M_3X > MX_2$

4.(D) mmol of base $= 2.5\times\frac{2}{5} = 1$

mmol of acid required to reach the end point = 1

Volume of acid required to reach the end point $= \frac{15}{2}\text{mL}$

Total volume at the end point $= \frac{15}{2} + 2.5 = 10\,\text{mL}$

Molarity of salt at the end point $= \frac{1}{10} = 0.10$

$\underset{C(1-\alpha)}{B^+} + H_2O \rightleftharpoons \underset{Ca}{BOH} + \underset{Ca}{H^+}$

$K_h = \frac{K_w}{K_b} = 10^{-2}$

$K_h = 10^{-2} = \frac{Ca}{1-a} = \frac{0.1a}{1-a} \quad \Rightarrow 10a^2 + a - 1 = 0$

$\Rightarrow a = \frac{-1+\sqrt{1+40}}{20} = 0.27 \quad \Rightarrow [H^+] = Ca = 0.1\times0.27 = 0.027M$

SECTION 2 | MULTIPLE CORRECT ANSWERS TYPE

5.(AB) Azeotropic mixture can not be separated into components by distillation.

6.(BC) (A) Frenkel defect is favoured when the difference in ionic radii is very large. In the given case Schottky defect is favoured. Hence option (A) is wrong.

(B) Frenkel defect arises due to dislocation of smaller ion from its normal lattice point to the interstitial space. Hence option (B) is correct.

(C) When alkali metal in the gaseous state is passed over salts like NaCl, electrons from metal are trapped in the interstitials space giving rise to F-centre defect which is a type of Frenkel defect. Hence option (C) is correct.

(D) Schottky defect affects physical properties of solids. Hence option (D) is wrong.

7.(BC) The reason for temporary hardness is owing to the presence of bicarbonates of Ca and Mg. To remove this temporary hardness, we have to use agents that can precipitate out the bicarbonates in an easy-to-filter-out form :

$$Ca(HCO_3)_2 + Na_2CO_3 \rightarrow CaCO_3 \downarrow + NaHCO_3$$

$$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3 \downarrow + 2H_2O$$

8.(AD) Chemisorption is unimolecular layer and exothermic.
The enthalpy change in Physisorption is 20 - 40 kJ/mol.
Physisorption decreases with increase of temperature.

9.(AB) Benzene + toluene will form ideal solution
Phenol + aniline will show negative deviation

10.(BCD) $\Delta S_{system} > 0$

$\Delta S_{surrounding} = 0$

$\Delta H_{mixing} = 0$

SECTION 3 | SINGLE DIGIT INTEGER TYPE

1.(5)

2.(6) $H_2SO_4 \longrightarrow H^+ + HSO_4^-$

1 1

$Na_2SO_4 \longrightarrow 2Na^+ + SO_4^{2-}$

0 0.036 0.018

$HSO_4^- \rightleftharpoons H^+ + SO_4^{2-}$

1 1 0.018

($Q > K_C$ so the reaction will go in backward direction)

$(1+a) \quad (1-a)(0.018-a)$

$\approx 1 \quad \approx 1$

Now, $0.012 = \dfrac{1 \times (0.018 - a)}{1}$

a = 0.006

$\left[SO_4^{2-}\right] = 0.018 - 0.006 = 1.2 \times 10^{-2} M$

$PbSO_4(s) \rightleftharpoons Pb^{2+}(aq) + SO_4^{2-}(aq)$

S $(0.012 + s) \approx 0.012$

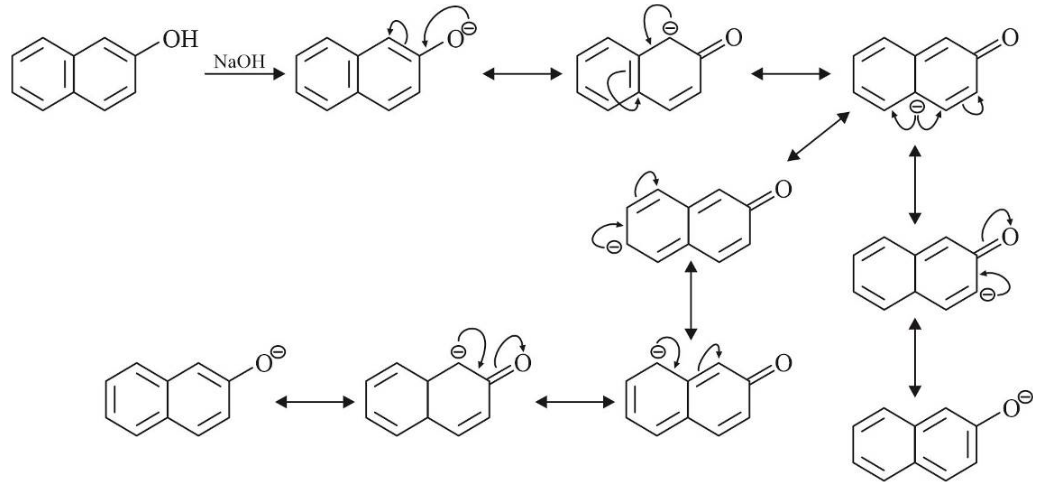
$$s = \frac{1.6 \times 10^{-8}}{0.012} = 1.33 \times 10^{-6} \Rightarrow \therefore y = 6$$

3.(0)

$a - x$	0.75	0.4	0.1
Δx	-0.25	-0.6	-0.9
$rate = -\dfrac{\Delta x}{\Delta t}$	$-\dfrac{-0.25}{0.05} = 5$	$-\dfrac{-0.6}{0.12} = 5$	$-\dfrac{-0.9}{0.18} = 5$

Since rate $= -\dfrac{\Delta x}{\Delta t}$ = constant for different concentration values, so the reaction is of zero order.

4.(9)



5.(6) The formula for conductance is $G = \kappa \times \frac{a}{\ell}$

$$5 \times 10^{-7} = \kappa \times \frac{1}{120}$$

$$\kappa = 6 \times 10^{-5} \, s\,cm^{-1}$$

$$\wedge_m^c = \frac{\kappa \times 1000}{M} = \frac{6 \times 10^{-5} \times 1000}{0.0015} = 40$$

$$\because \; pH = 4$$

$$\therefore \; [H^+] = 10^{-4} = c\alpha = 0.0015\alpha$$

$$\alpha = \frac{10^{-4}}{0.0015}$$

$$\alpha = \frac{\wedge_m^c}{\wedge_m^0} \Rightarrow \frac{10^{-4}}{0.0015} = 40$$

$$\wedge_m^0 = 6 \times 10^2 s \; cm^2 \; mole^{-1}$$

$z = 6$

6.(5) $\frac{P_0 - P_s}{P^0} = iX_B$

$$\frac{60 - 59.724}{60} = i \times \frac{0.1}{0.1 + 100}$$

$$\frac{0.276}{60} = i \times \frac{0.1}{100.1} = i \times \frac{1}{1000}$$

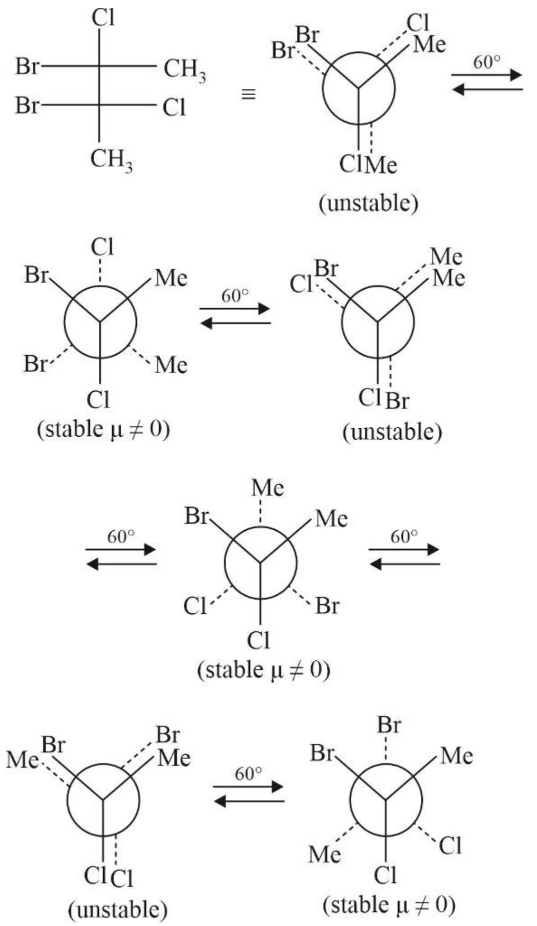
$$i = \frac{276}{60} = 4.6$$

$$i = 1 + (n-1)a$$

$$4.6 = 1 + (n-1)0.9$$

$$n = 5$$

7.(3)



8.(1) $Mg(s) + Cu^{2+}(aq) \longrightarrow Mg^{2+}(aq) + Cu(s)$

$E^0_{cell} = 2.70 \quad E_{cell} = 2.67 \quad Mg^{2+} = xM$

$Cu^{2+} = 1M$

$$E_{cell} = E^0_{cell} - \frac{RT}{nF}\ln x$$

$$2.67 = 2.70 - \frac{RT}{2F}\ln x$$

$$-0.03 = -\frac{R \times 300}{2F} \times \ln x$$

$$\ln x = \frac{0.03 \times 2}{300} \times \frac{F}{R}$$

$$= \frac{0.03 \times 2 \times 11500}{300 \times 1}$$

$$\ln x = 2.30 = \ln(10)$$

$$x = \frac{10}{10} = 1$$

MATHEMATICS

SECTION - 1 | SINGLE CHOICE CORRECT TYPE

1.(B) $\left|\int_a^b f(x)dx-(b-a)f(a)\right|=\left|\int_a^b f(x)dx-\int_a^b f(a)dx\right|=\left|\int_a^b (f(x)-f(a))dx\right|\le\int_a^b |f(x)-f(a)|dx$

$\le\int_a^b |x-a|dx=\int_a^b (x-a)dx=\frac{(b-a)^2}{2}$

2.(D) $f'(x)=\frac{192x^3}{2+\sin^4(\pi x)}\ \forall x\in R\ ;\ f\left(\frac{1}{2}\right)=0$

Now, $64x^3\le f'(x)\le 96x^3\ \forall x\in\left[\frac{1}{2},1\right]$

$\therefore\ \int_{1/2}^{x}64x^3dx\le\int_{1/2}^{x}f'(x)dx\le\int_{1/2}^{x}96x^3dx$

$16x^4-1\le f(x)\le 24x^4-\frac{3}{2}\ \forall x\in\left[\frac{1}{2},1\right]$

$\therefore\ \int_{1/2}^{1}(16x^4-1)dx\le\int_{1/2}^{1}f(x)dx\le\int_{1/2}^{1}\left(24x^4-\frac{3}{2}\right)dx$

$\therefore\ \frac{16}{5}\cdot\frac{31}{32}-\frac{1}{2}\le\int_{1/2}^{1}f(x)dx\le\frac{24}{5}\cdot\frac{31}{32}-\frac{3}{4}$

$\Rightarrow\ \frac{26}{10}\le\int_{1/2}^{1}f(x)dx\le\frac{39}{10}$

3.(B) The point that divides $5\hat{i}$ & $5\hat{j}$ in the ratio of

$k:1$ is $\frac{(5\hat{j})k+(5\hat{i})1}{k+1}$

$\therefore\ \vec{b}=\frac{5\hat{i}+5k\hat{j}}{k+1}$

Also, $|\vec{b}|\le\sqrt{37}$

$\Rightarrow\ \frac{1}{|k+1|}\sqrt{25+25k^2}\le\sqrt{37}$

$5\sqrt{1+k^2}\le\sqrt{37}\,|k+1|$

Squaring both sides, we get

$25(1+k^2)\le 37(k^2+2k+1)$

$6k^2+37k+6\ge 0$ or $(6k+1)(k+6)\ge 0$

$k \in (-\infty, -6] \cup \left[-\frac{1}{6}, \infty\right)$

4.(D) A plane containing the line of intersection of the given planes is

$x - y - z - 4 + \lambda(x + y + 2z - 4) = 0$

$(\lambda + 1)x + (\lambda - 1)y + (2\lambda - 1)z - 4(\lambda + 1) = 0$

vector normal to it

$V = (\lambda + 1)\hat{i} + (\lambda - 1)\hat{j} + (2\lambda - 1)\hat{k}$ …(i)

Now the vector along the line the intersection of the planes $2x + 3y + z - 1 = 0$ and $x + 3y + 2z - 2 = 0$ is given by

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix} = 3(\hat{i} - \hat{j} + \hat{k})$$

As $\vec{n}$ is parallel to the plane (i), we have

$\vec{n} \cdot \vec{V} = 0$

$(\lambda + 1) - (\lambda - 1) + (2\lambda - 1) = 0$

$2 + 2\lambda - 1 = 0$ or $\lambda = \frac{-1}{2}$

Hence, the required plane is

$\frac{x}{2} - \frac{3y}{2} - 2z - 2 = 0$

$x - 3y - 4z - 4 = 0$

Hence, $|A + B + C| = 11$

SECTION 2 | MULTIPLE CORRECT ANSWERS TYPE

5.(ABC) $f(\alpha) = \lim_{x \to 2} \left(\sin^x \alpha + \cos^x \alpha\right)^{\frac{1}{(x-2)}}$ (1^∞ form)

$$= \begin{cases} e^{\lim_{x \to 2} \frac{\sin^x \alpha + \cos^x \alpha - 1}{x - 2}}, & \alpha \in \left(0, \frac{\pi}{2}\right) \\ 1, & \alpha = 0, \frac{\pi}{2} \end{cases}$$

Now, $e^{\lim_{x \to 2} \frac{\sin^x \alpha + \cos^x \alpha - 1}{x-2}} = e^{\lim_{x \to 2} \frac{\sin^x \alpha + \cos^x \alpha - \sin^2 \alpha - \cos^2 \alpha}{x-2}}$

$= e^{\lim_{x \to 2} \frac{\sin^x \alpha(\sin^{x-2} \alpha - 1) + \cos^x \alpha(\cos^{x-2} \alpha - 1)}{x-2}}$

$= e^{\sin^2 \alpha \log_e \sin \alpha + \cos^2 \alpha \log_e \cos \alpha}$

$= e^{\log_e (\sin \alpha)^{\sin^2 \alpha} + \log_e (\cos \alpha)^{\cos^2 \alpha}}$

$$= e^{\log_e (\sin\alpha)^{\sin^2\alpha}(\cos\alpha)^{\cos^2\alpha}}$$

$$= (\sin\alpha)^{\sin^2\alpha}(\cos\alpha)^{\cos^2\alpha}$$

$$\therefore\ f(x) = \begin{cases} (\cos\alpha)^{\cos^2\alpha}\cdot(\sin\alpha)^{\sin^2\alpha}, & \alpha\in\left(0,\frac{\pi}{2}\right) \\ 1, & \alpha = 0, \frac{\pi}{2} \end{cases}$$

6.(BD) We have $g = \frac{1}{f}$

$$\therefore\ g' = \frac{-1}{f^2}f'$$

$$g'' = -\left[-\frac{2}{f^3}f'^2 + \frac{1}{f^2}f''\right]$$

$$= \frac{2}{f^3}f'^2 - \frac{f''}{f^2}$$

$$\frac{f''}{f'} - \frac{g''}{g'} = \frac{f''}{f'} - \frac{\frac{2}{f^3}f'^2 - \frac{f''}{f^2}}{-\frac{1}{f^2}f'}$$

$$= \frac{f''}{f'} - \left(\frac{-2f'}{f} + \frac{f''}{f'}\right) = \frac{2f'}{f}$$

Also, $g\cdot f = 1$

$$g'f + gf' = 0$$

$$\therefore\ \frac{f''}{f'} - \frac{g''}{g'} = -\frac{2g'}{g}$$

7.(AB) $y = e^{-x}\cos x$

$$y_1 = -e^{-x}\cos x - e^{-x}\sin x = \sqrt{2}e^{-x}\cos\left(x - \frac{\pi}{4}\right)$$

$$y_2 = \left(-\sqrt{2}\right)^2 e^{-x}\cos\left(x - \frac{\pi}{2}\right)$$

$$y_3 = \left(-\sqrt{2}\right)^3 e^{-x}\cos\left(x - \frac{3\pi}{4}\right)$$

$$y_4 = \left(-\sqrt{2}\right)^4 e^{-x}\cos(x-\pi) = -4e^{-x}\cos x$$

$y_4 + 4y = 0$ or $k_4 = 4$

Differentiating it again four times, we get

$$y_8 + 4y_4 = 0$$

$y_8 - 16y = 0$

$k_8 = -16$

Further $y_{12} + 4y_8 = 0$

$y_{12} + 64y = 0$

$k_{12} = 64$

Similarly, $k_{16} = -256$

8.(ABCD) Given function is discontinuous when $a + \sin \pi x = \pm 1$

Now, if $a = 1$, then $\sin \pi x = 0$ or $x = 1, 2, 3, 4, 5$

If $a = 3$, then $\sin \pi x = -2$, not possible

If $a = 0.5$, then $\sin \pi = 0.5$

Therefore, x has 6 values, 2 each for one cycle of period 2

If $a = 0$, then $\sin \pi x = \pm 1$ or $x = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}$

Hence all the options are correct

9.(ABD) We have $0 < x < 1$

$\Rightarrow \ 0 < 2^n x < 2^n$

$\Rightarrow \ [2^n x]$ is discontinuous when $2^n x = 1, 2, 3, ..., 2^n - 1$

$\Rightarrow \ \{2^m x\}$ is discontinuous when $2^m x = 1, 2, 3, ..., 2^m - 1$

But $(2^m - 1)$ number of points are common where both are discontinuous but their sum is continuous

Hence total number of points of discontinuity

$(2^n - 1) + (2^m - 1) - 2(2^m - 1) = 2^n - 2^m (n > m)$

$2^n - 2^m = 24$, for $n = 5, m = 3$

$2^n - 2^m = 28$, for $n = 5, m = 2$

$2^n - 2^m = 496$, for $n = 9, m = 4$

10.(ABCD) (A) Let $f(x) = e^x \cos x - 1$

$\therefore \ f'(x) = e^x(\cos x - \sin x) = 0$

$\therefore \ \tan x = 1$, which has a root between two roots of $f(x) = 0$

(B) Let $f(x) = e^x \sin x - 1$

$\therefore \ f'(x) = e^x(\sin x + \cos x) = 0$

$\therefore \ \tan x = -1$, which has a root between two roots of $f(x) = 0$

(C) Let $f(x) = e^{-x} - \cos x$

$\therefore \ f'(x) = -e^{-x} + \sin x = 0$

$\therefore \ e^{-x} = \sin x$, which has a root between two roots of $f(x) = 0$

(D) Let $f(x)=e^{-x}-\sin x$

$\therefore\ f'(x)=-e^{-x}-\cos x=0$

$\therefore\ e^{-x}=-\cos x,$ which has a root between two roots of $f(x)=0$

SECTION 3 | SINGLE DIGIT INTEGER TYPE

1.(6) We have, $\lim_{x\to 2}\frac{\sqrt[3]{60+x^2}-\sqrt[3]{64}}{\sin(x-2)}$

$$=\lim_{x\to 2}\frac{\left[(60+x^2)^{\frac{1}{3}}-64^{\frac{1}{3}}\right]\left[(60+x^2)^{\frac{2}{3}}+(60+x^2)^{\frac{1}{3}}64^{\frac{1}{3}}+64^{\frac{2}{3}}\right]}{(x-2)\frac{\sin(x-2)}{(x-2)}\left[(60+x^2)^{\frac{2}{3}}+(60+x^2)^{\frac{1}{3}}64^{\frac{1}{3}}+64^{\frac{2}{3}}\right]}$$

$$=\lim_{x\to 2}\frac{60+x^2-64}{(x-2)[16+4\times 4+16]}$$

$$=\lim_{x\to 2}\frac{(x-2)(x+2)}{48(x-2)}$$

$$=\lim_{x\to 2}\frac{x+2}{48}=\frac{4}{48}=\frac{1}{12}$$

2.(6) Clearly, n is even. Then,

$$=\lim_{n\to\infty}\left(2^{1+3+5+....+n/2\ terms}\cdot 3^{2+4+6+....+n/2\ terms}\right)^{\frac{1}{(n^2+1)}}$$

$$=\lim_{n\to\infty}\left(2^{\frac{n^2}{4}}\cdot 3^{\frac{n(n+2)}{4}}\right)^{\frac{1}{(n^2+1)}}$$

$$=\lim_{n\to\infty}2^{\frac{n^2}{4(n^2+1)}}\cdot 3^{\frac{n(n+2)}{4(n^2+1)}}$$

$$=2^{\lim_{n\to\infty}\frac{1}{4\left(1+\frac{1}{n^2}\right)}}\cdot 3^{\lim_{n\to\infty}\frac{\left(1+\frac{2}{n}\right)}{4\left(1+\frac{1}{n^2}\right)}}$$

$$=2^{\frac{1}{4}}3^{\frac{1}{4}}=(6)^{\frac{1}{4}}$$

3.(7) $$\lim_{x\to 0}\frac{x^2\left\{\beta x-\frac{(\beta x)^3}{3!}+....\right\}}{\alpha x-\left(x-\frac{x^3}{3!}+....\right)}=1$$

$$\Rightarrow \lim_{x\to 0}\frac{x^3\left(\beta-\frac{\beta^3x^2}{3!}+....\right)}{(\alpha-1)x+\frac{x^3}{3!}-\frac{x^5}{5!}+....}=1$$

$$\Rightarrow \lim_{x\to 0}\frac{x^2\left(\beta-\frac{\beta^3x^2}{3!}+....\right)}{(\alpha-1)+\frac{x^2}{3!}-\frac{x^4}{5!}+....}=1$$

$\alpha-1=0$ or $\alpha=1$

$$\Rightarrow \lim_{x\to 0}\frac{\beta-\frac{\beta^3}{3}x^2....}{\frac{1}{3!}-\frac{x^2}{5!}+....}=1$$

$$\therefore\ \beta=\frac{1}{3!}=\frac{1}{6}$$

$$\therefore\ 6(\alpha+\beta)=6\left(1+\frac{1}{6}\right)=7$$

4.(9) $\frac{d}{dx}\{[(f(x)]^2-[\phi(x)]^2\}$

$=2[f(x)\cdot f'(x)-\phi(x)\cdot\phi'(x)]$

$=2[f(x)\cdot\phi(x)-\phi(x)\cdot f(x)]$ $[\because\ f'(x)=\phi(x)$ and $\phi'(x)=f(x)]$

$=0$

$[f(x)]^2-[\phi(x)]^2=$ constant

$\therefore\ [f(10)]^2-[\phi(10)]^2=[f(3)]^2-[\phi(3)]^2=[f(3)]^2-[f'(3)]^2$

$=25-16=9$

5.(6) $f(x)=(x^3+bx^2+cx+d)|x-1||x-2|(x-3)^2|x-4|(x-4)^2$

$f(x)$ is differentiable for all real x, if $(x-1)$ and $(x-2)$ are factors of $g(x)$

$f'''(4)$ exists, for which $(x-4)$ must be factor of $g(x)$

$\therefore\ g(x)=x^3+bx^2+cx+d=(x-1)(x-2)(x-4)$

$g(5)=4\times3\times1=12$

6.(8) $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{f(x)+f(h)+2xh(x+h)-\frac{1}{3}-\left(f(x)+f(0)-\frac{1}{3}\right)}{h}$$

$$=\lim_{h\to 0}\frac{f(h)-f(0)}{h}+2x^2$$

$= f'(0) + 2x^2$

Now, $\lim_{h\to 0} \frac{3f(h)-1}{6h} = \lim_{h\to 0} \frac{f(h)-\frac{1}{3}}{2h} = \lim_{h\to 0} \frac{f(h)-f(0)}{2h}$

$= \frac{f'(0)}{2} = \frac{2}{3}$

$\therefore \ f'(0) = \frac{4}{3}$ $\qquad \therefore \ f'(x) = \frac{4}{3} + 2x^2$

$f(x) = \lambda + \frac{4}{3}x + \frac{2x^3}{3}$ or $f(0) = \lambda = \frac{1}{3}$ $\qquad \therefore \ f(x) = \frac{2x^3}{3} + \frac{4}{3}x + \frac{1}{3}$ or $f(2) = \frac{25}{3}$

7.(4) We have $f(0) = 2$

Now, $y - f(a) = f'(a)[x-a]$

For x intercept, $y = 0$. So,

$x = a - \frac{f(a)}{f'(a)} = a - 2$ or $\frac{f(a)}{f'(a)} = 2$

$\frac{f'(a)}{f(a)} = \frac{1}{2}$

$\therefore$ On integrating both sides w.r.t. a, we get

In $f(a) = \frac{a}{2} + \log_e C$

$f(a) = Ce^{a/2}$

$f(x) = Ce^{x/2}$

$f(0) = C$ or $C = 2$

$\therefore \ f(x) = 2e^{x/2}$

Hence, $k = 2,\ p = \frac{1}{2}\ or\ \frac{k}{p} = 4$

8.(3) Let $y = 2x\tan^{-1}x - \ln(1+x^2)$

$y' = 2\tan^{-1}x + \frac{2x}{1+x^2} - \frac{2x}{1+x^2}$

$\therefore \ y' > 0\, \forall x \in R^+,\ y' < 0\ \forall\, x \in R^-$

S is the set of all real numbers except zero.